Die Qual der Wahl

wieso wir in Entscheidungssituationen mehr Fantasie brauchen

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Lehrstuhl für Operations Research
RWTH Aachen University

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@mluebbecke

26. September 2008

Ich denke, dass ein anwendungsnaher, in der Optimierung beheimateter Mathematiker mehr als nur eine solide Ergänzung Ihrer Kompetenzen darstellt. Ich würde mich daher sehr darüber freuen, mich und meine Arbeit bei Ihnen vorstellen zu dürfen.

Mit freundlichem Gruß

Agenda

Project

Problem

Progress

Proofs

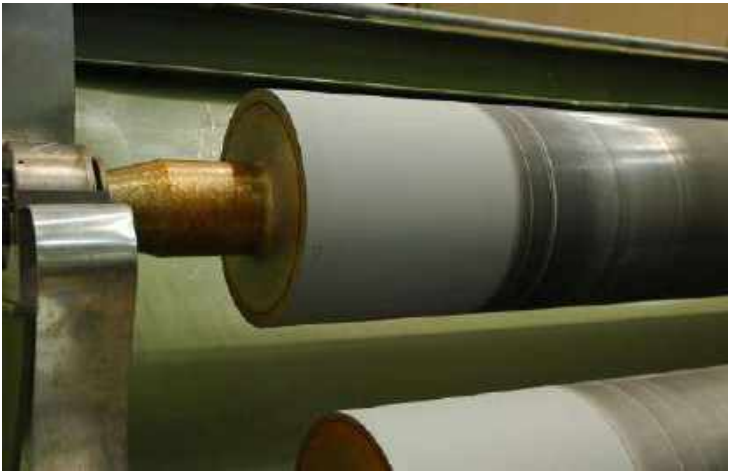
Profits

Steel Production

Coil Coating

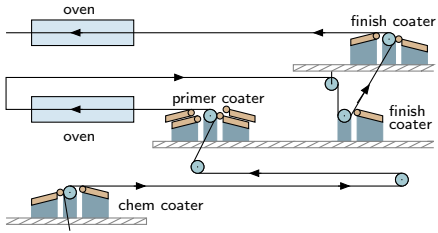
Coil Coating: Local Cost

Coil Coating: Global Cost



Coil Coating: A Sequencing Problem?

A sequencing problem...



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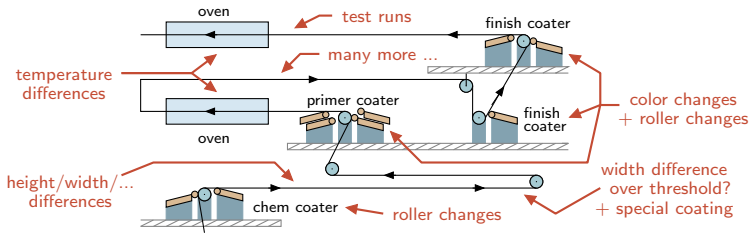

with a very complicated cost function.

Shuttle Coaters

Tank Assignment Problem

Given a fixed sequence of coils

Tank Assignment Problem

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- color changes $\rightsquigarrow$ cleaning

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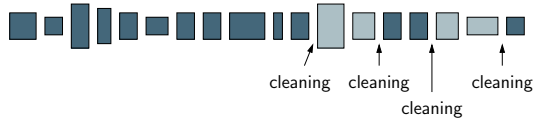

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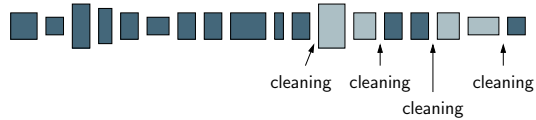

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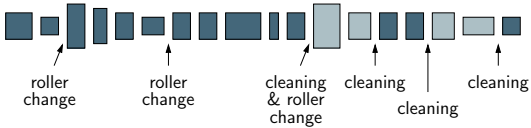

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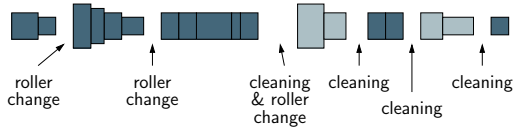

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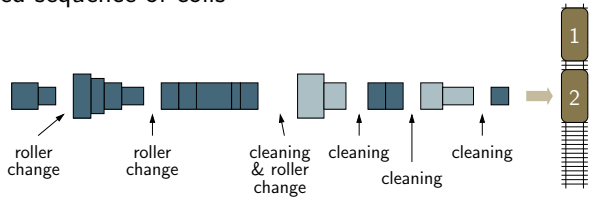

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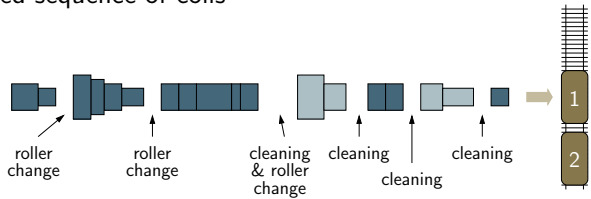

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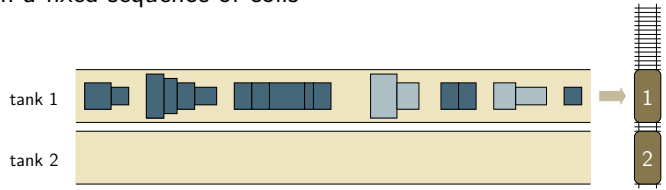

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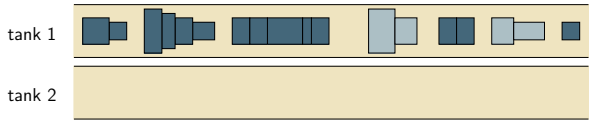

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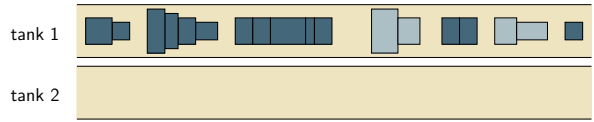

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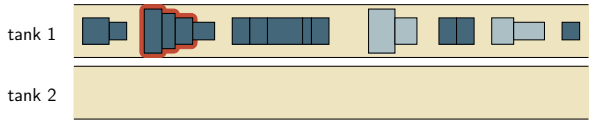

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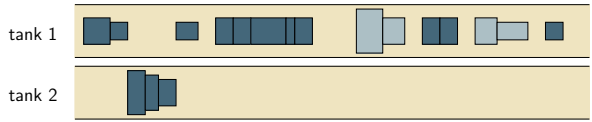

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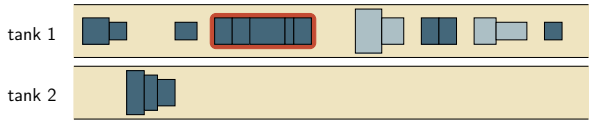

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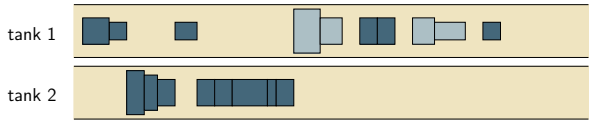

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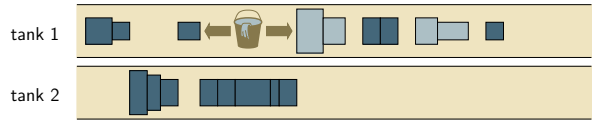

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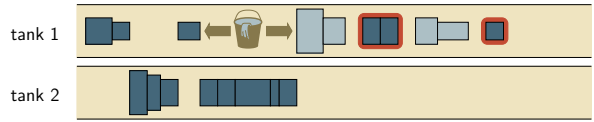

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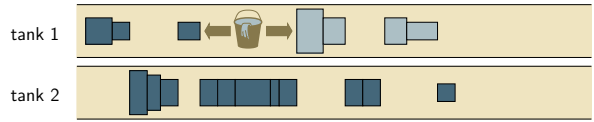

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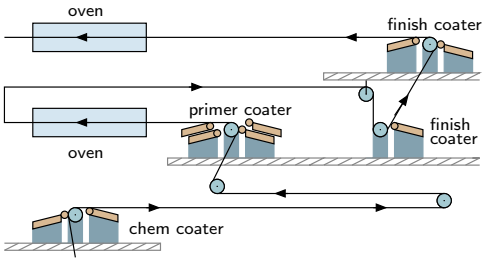

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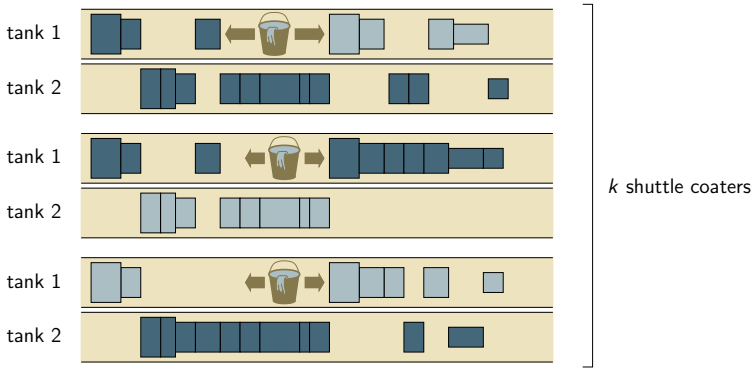

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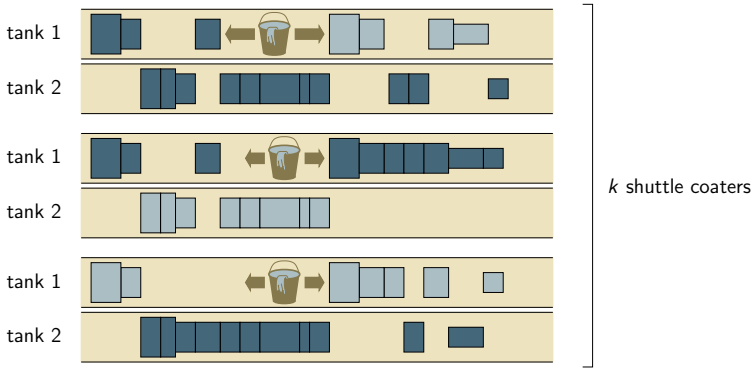

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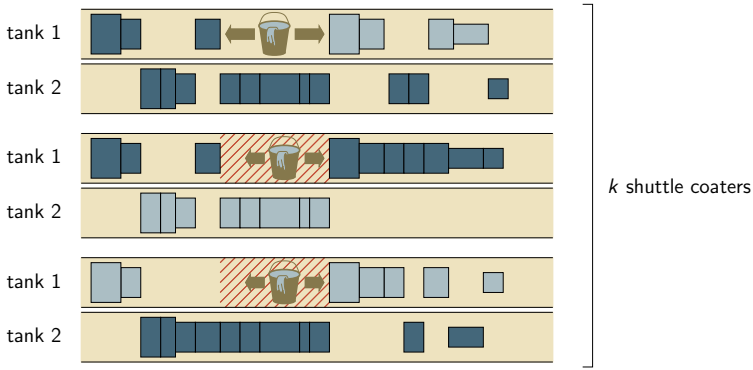

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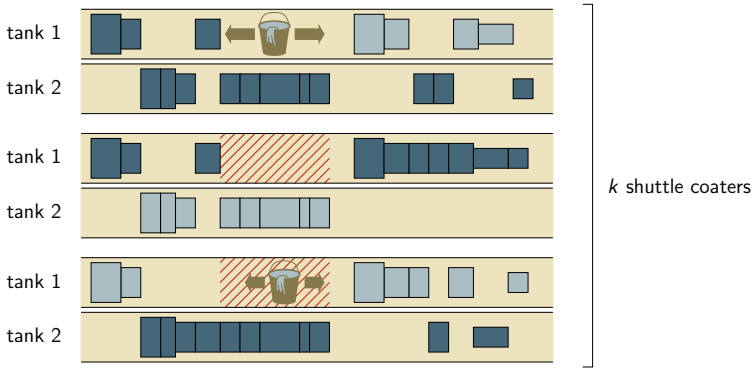

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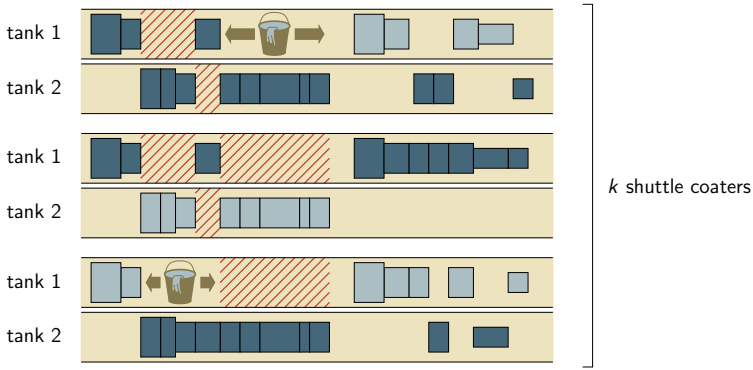

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An dieser Stelle . . .

. . . wäre jetzt die Mathematik

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Was ist die Aufgabe?

- gegeben eine Liste von zu beschichtenden Coils mit ihren Eigenschaften
- bestimme eine Reihenfolge für alle Coils (inkl. Farb- und Wannenbelegung)

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- wenn ich „optimal“ sage, meine ich „unverbesserbar“, „bestmöglich“
- wenn ich „optimieren“ sage, meine ich das Unverbesserbare zu erreichen

Mathematische Optimierung

- Auffinden einer beweisbar bestmöglichen Lösung unter Nebenbedingungen

mathematisches Optimierungsmodell

- erfasst knapp und präzise *alle möglichen* Lösungen
  // in unserem Fall sind das alle erlaubten Reihenfolgen
- deklarativ, nicht prozedural

→ man beschreibt, wie das Endergebnis auszusehen hat, nicht wie man es erzielt

- die Mathematik hat Algorithmen, die Optimallösungen garantieren
- man erhält *immer* auch Gütegarantien (USP der Math. Opt.)

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Visualization/Verification of Solutions

At a glance: Sequence, tank assignment, and cleaning schedule

+ a detailed Excel sheet

Was hat es gebracht?

- etwa 30% weniger Leerzeiten ggü. früheren „händischen“ Plänen
- weniger als 10% verbleibendes Einsparpotenzial bei Durchlaufzeiten
- es wurden prinzipiell neue (Ideen für) Reihenfolgen gefunden

Agenda

Project Lessons Learned: Our Expectation ...

Engineering Team

Technology & Business Case

Nick
Wirt.-Ing. MB

Jiangyue
Wirt.-Ing. WPT

Hermann
Wirt.-Ing. MB

Modeling Team

Mathematical formulation

Lennart
Wirt.-Ing. ET

Katrin
Wirt.-Ing. MB

Coding Team

Programming and implementation

Sai
DDS

Gina
CS

Viraj
DDS

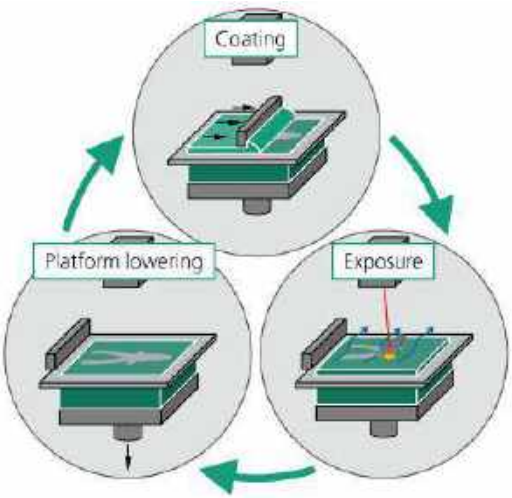
Contact

Elisabeth | Marco

rodriguez-heck@or.rwth-aachen.de

luebbecke@or.rwth-aachen.de

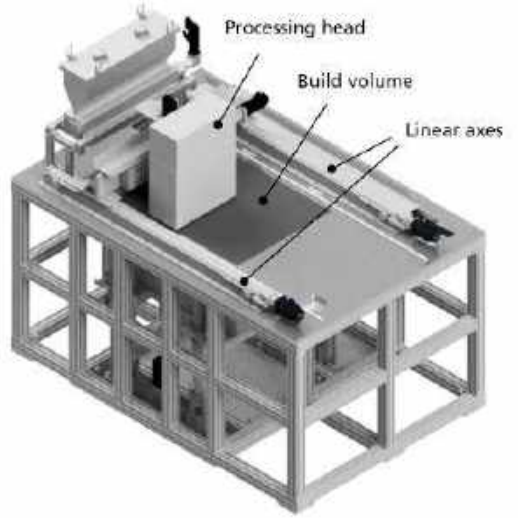
Laser Powder Bed Fusion (LPBF)



Source: Fraunhofer ILT

- Additive manufacturing technology which allows to 3D-print metal parts of highest complexity
- 3D-CAD-parts are sliced into several layers which are then laid-up one by one in a repetitive process cycle
- Involves the exposure of metal powder to a high power-density laser beam to create a solid metal part
- Applications in variety of domains
  - Aerospace
  - Automotive
  - Medical
  - ...

Scalable LPBF machine concept

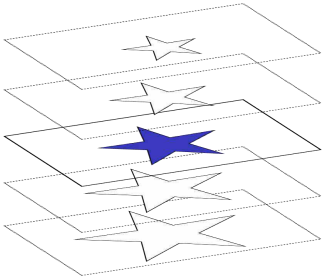


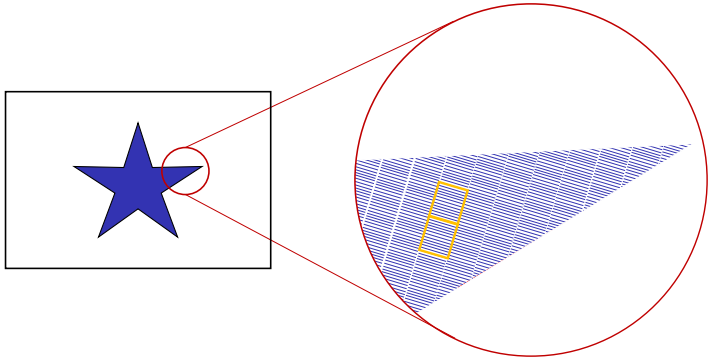
Source: Fraunhofer ILT

- Movable processing head allows a cost-efficient scaling of the LPBF building volume
- Concept still calls for a versatile layer processing logic

$\Rightarrow$ Solution should decrease processing time per layer in order to reduce manufacturing costs

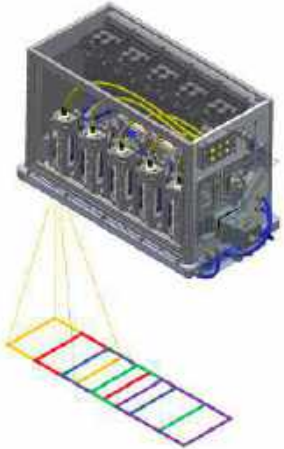
Processing layers

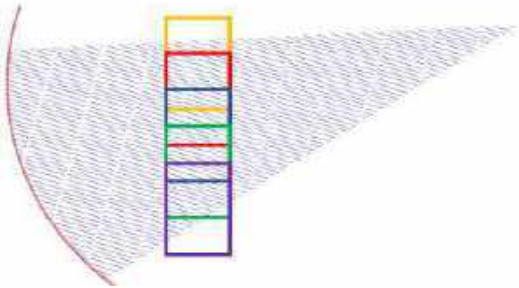




- Each layer consists of a set of given scan vectors which describe the part geometry
- Each scan vector is defined by two coordinates: (x_{start}, y_{start}), (x_{end}, y_{end})
- Scan vectors can be clustered into scan blocks

Multi-laser processing head





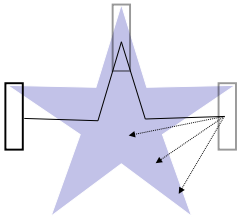
- Processing head can contain multiple scanners/lasers

$\Rightarrow$ Depending on the machine setup this leads to an overlap of scan fields

Distilling the problem

Challenge #1

Find a suitable path for the processing head to scan the whole geometry of the layer



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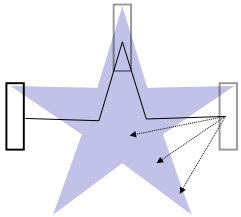
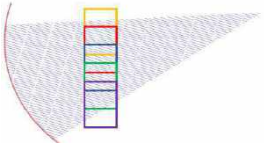


Challenge #2

Assign the given scan vectors/blocks to multiple scanners and sequence them for each scanner



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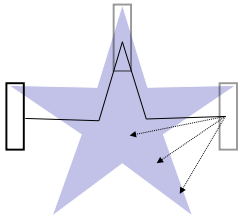

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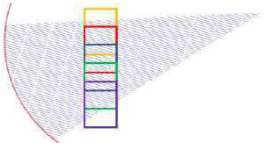
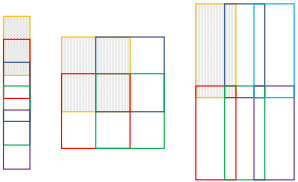


Challenge #3

Evaluate different machine setups for different geometries



Kurz innehalten: erster Gedanke

- „sinnvolle“ Szenarien/Konfigurationen/Pläne durchrechnen
- aus denen einen „besten“ auswählen

- verschwendet (massig) Potenzial
  // das ist aber erstmal niemandem vorzuwerfen!!

Kurz innehalten: zweiter Gedanke

- Daten, viele Daten
- Machine Learning („KI“)

- reproduziert die Vergangenheit // für manche Aufgaben gibt es keine Vergangenheit
- bringt möglicherweise Überraschendes hervor // aber eher zufällig

Kurz innehalten: unsere Studierenden fragen

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Kurz innehalten: unsere Studierenden fragen

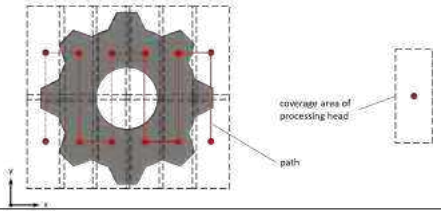
- ah! ein Überdeckungsproblem! und Machine Scheduling!
- klassische Standardprobleme des OR stecken hier drin

Grasping the problem from an OR perspective

Planning challenges

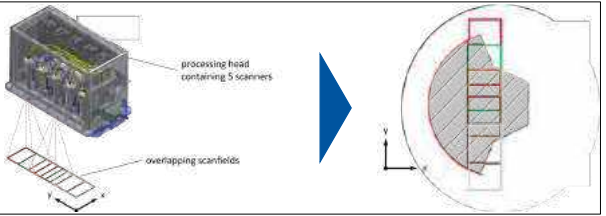
Challenge 1

- Determining a covering path for the processing head to scan layer geometry



Challenge 2

- Assignment of scan blocks to scanners with an ideal workload distribution



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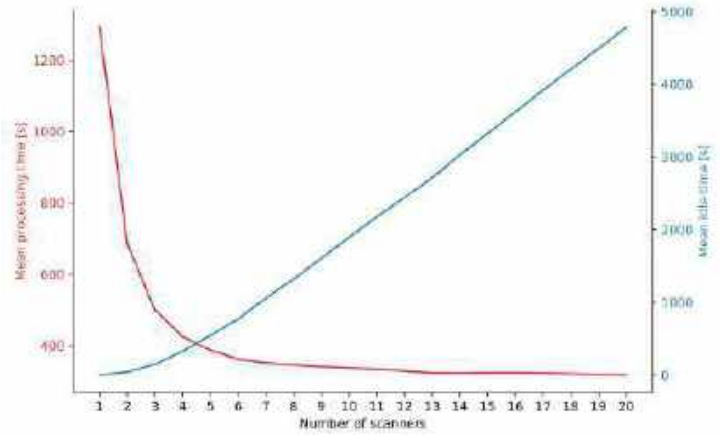
Testing the influence of different parameter settings

Number of scanners



- Significant reduction of processing time by almost 1000s from 1 to 6 scanners
- Curve flattens out and idle time increases when further increasing the number of scanners
- Adding scanners is particularly beneficial for large and dense geometries
- Sensitive decision due to the high cost of additional scanners

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Was macht die Mathematik hier „anders“?

- wie so oft sind „unzählige“ kleine Einzelentscheidungen zu treffen
  // kombinatorische Explosion
- von denen sehr viele voneinander abhängen und sich ggseitig beeinflussen
  // für Menschen unüberblickbar, weswegen „Daumenregeln“ so gut vermittelbar sind
- alle diese Entscheidungen werden bei uns gleichzeitig „im Konzert“ optimiert
  // wenn ein Computer plant, kann sich der Mensch auf das Wesentliche konzentrieren

Halten wir fest

- wir bauen Hardware auf Weltniveau
- das Niveau, auf dem wir sie betreiben ist dem oft nicht angemessen

  // plakativ, ich weiß, aber wir wollen ja ins Gespräch kommen ...

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Agenda

Echo

Aachener Nachrichten

Die Quadratur der Wahlkreise

Mathematiker der RWTH suchen eine Lösung für ein politisches Problem

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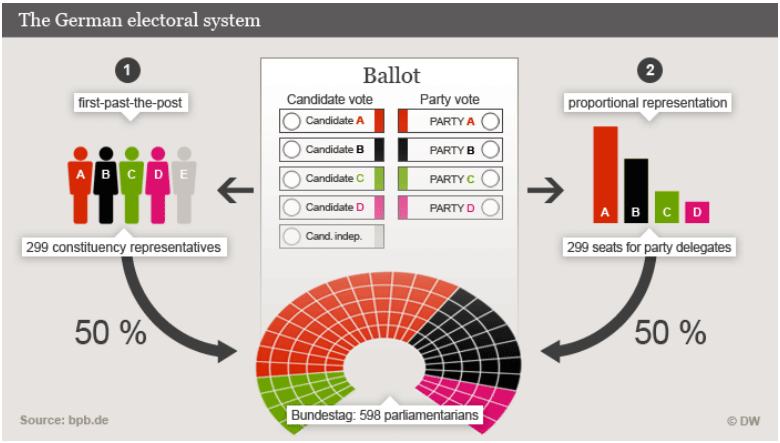
The German Parliament

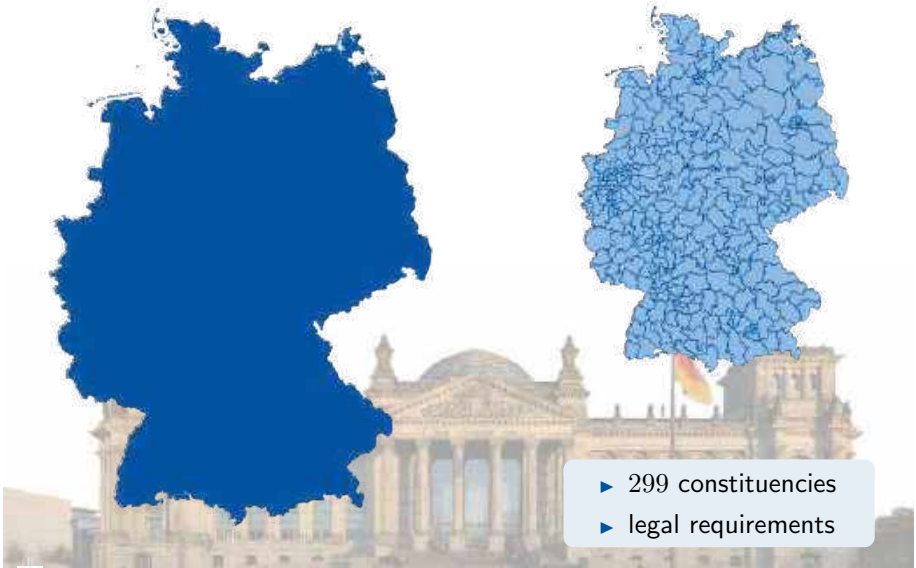
image source: wikipedia

709 seats, second largest parliament in the world

Electoral System in Germany

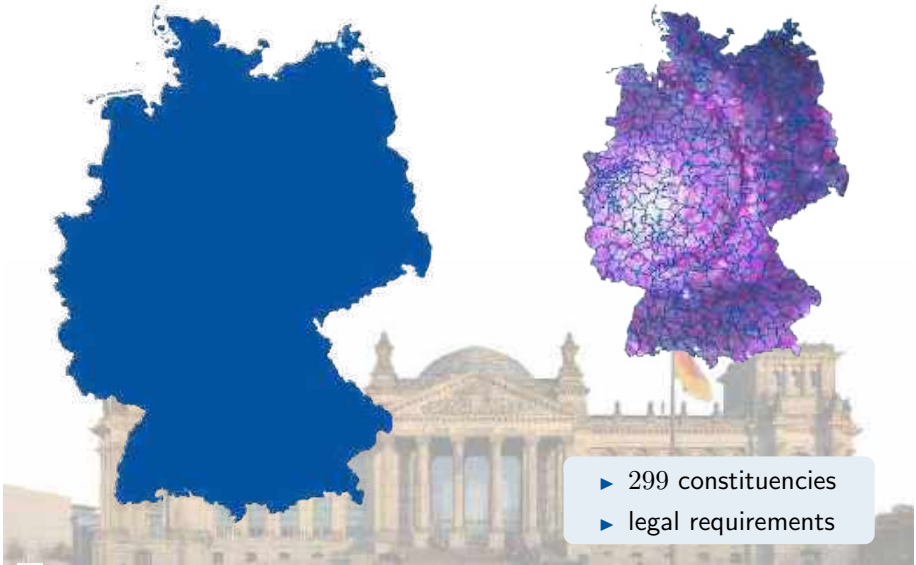


Mathematics optimally supports Political Districting



- 299 constituencies
- legal requirements

Mathematics optimally supports Political Districting



- 299 constituencies
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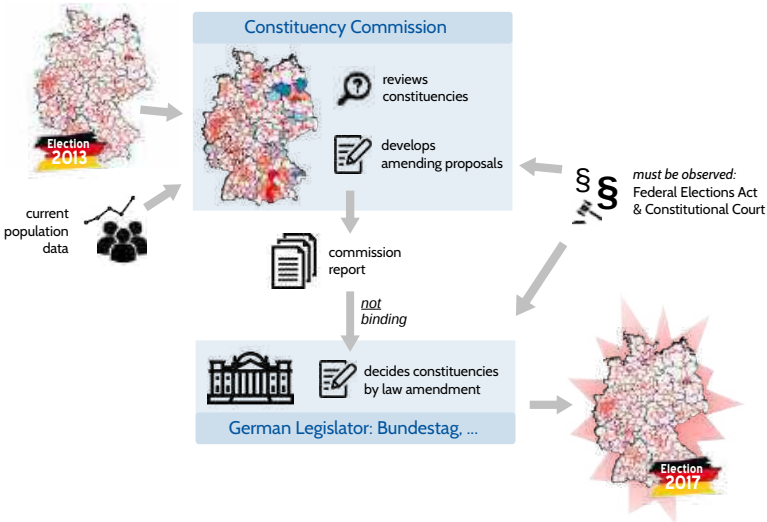
This Project started . . . in Class

image source: humsci.stanford.edu

Official Process of Adjusting the Constituencies



Legal Requirements and Principles on Constituencies

Per Federal State

Sec. 3 (1), No. 1–2 BWG

Distribution on Federal States via apportionment method

Conformity with administrative boundaries

Sec. 3 (1), No. 5 BWG

- governmental districts
- municipalities associations
- rural and urban districts
- municipalities, ...

Connectivity

Sec. 3 (1), No. 4 BWG

Electoral equality: population balance

Sec. 3 (1), No. 3 BWG

Two-stage deviation limit from average population of the constituencies

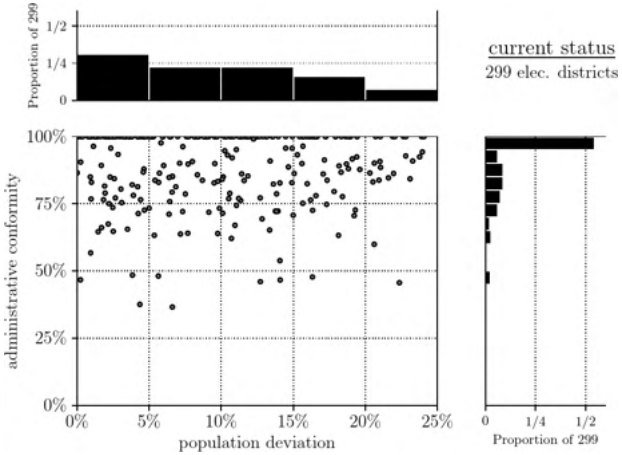
- tolerance limit: $\pm 15\%$
- maximum limit: $\pm 25\%$

Continuity

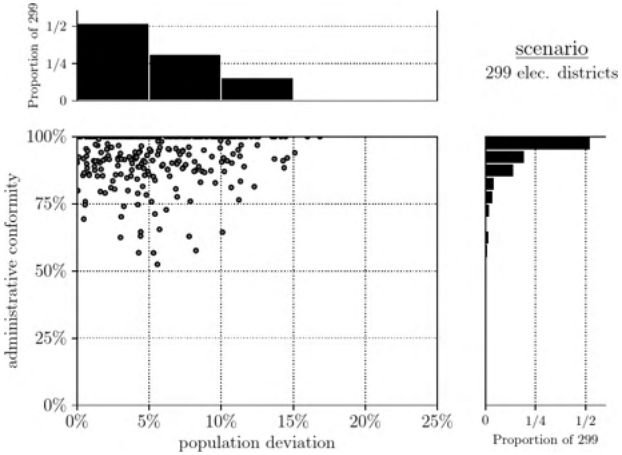
130, 212 and 95, 335 BVerfG

... in geographic layout of constituencies from one election to the next

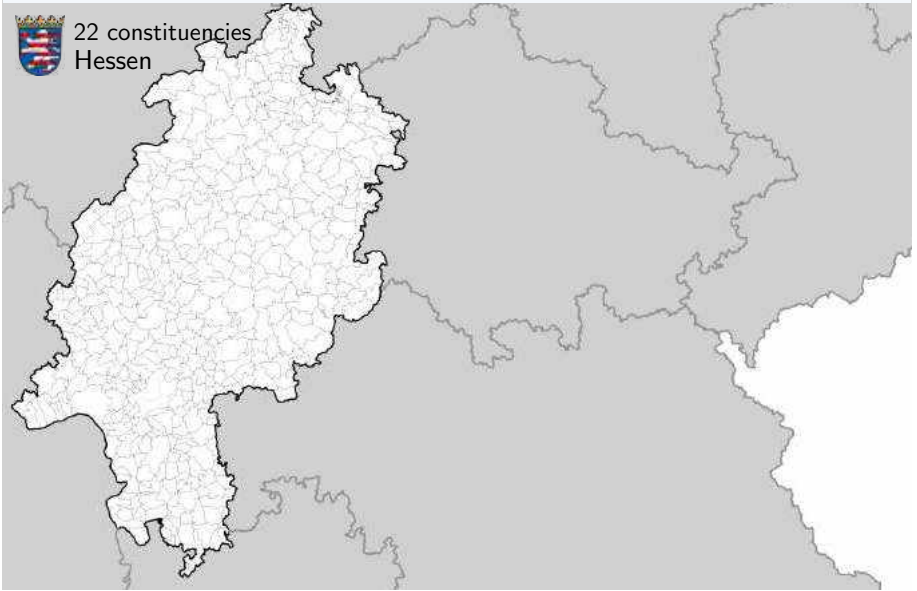
2017 Elections



2017 Elections



Network Model for the Political Districting Problem



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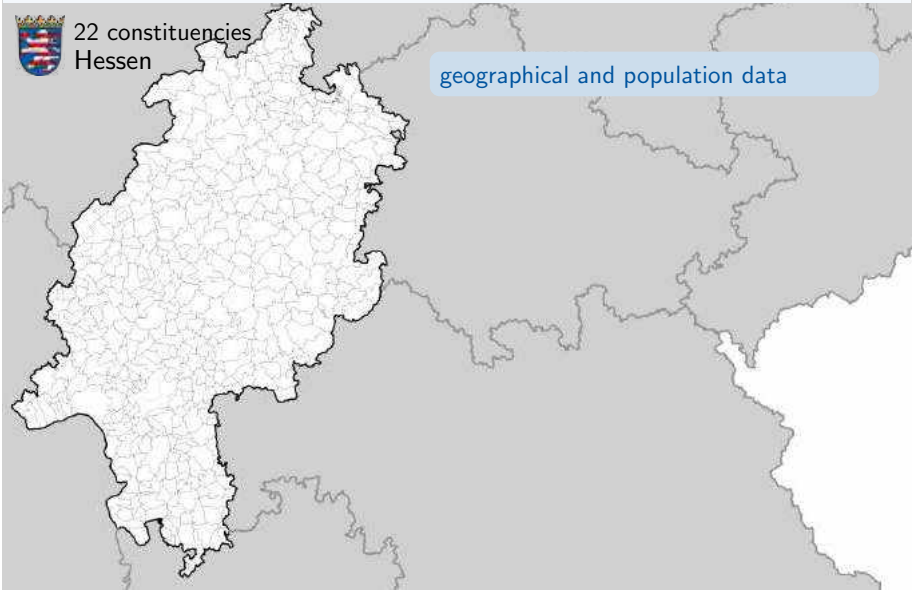

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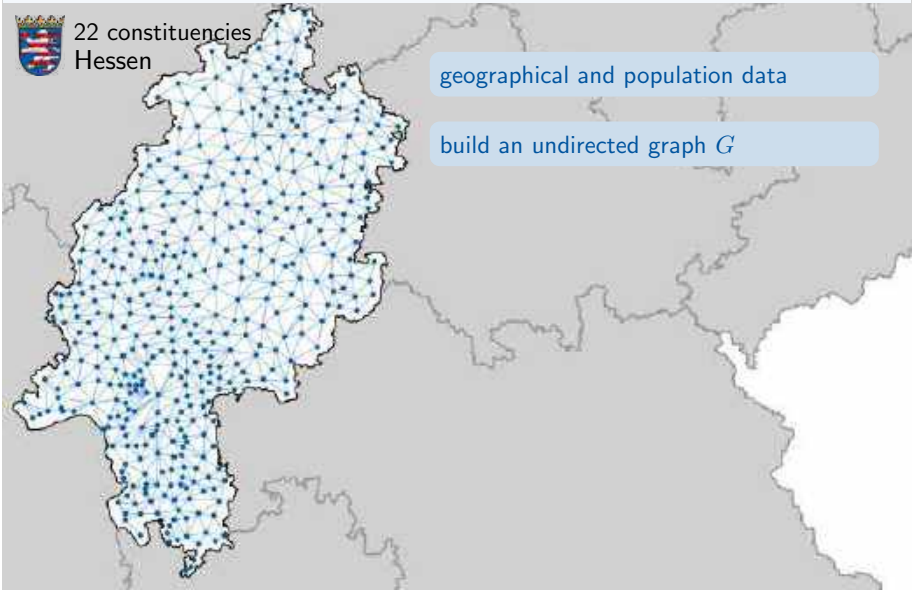
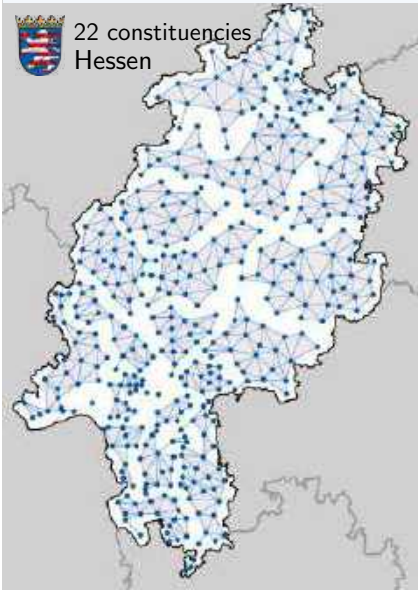


geographical and population data

build an undirected graph G

Network Model for the Political Districting Problem



22 constituencies
Hessen

geographical and population data

build an undirected graph G

optimally partition G into subgraphs

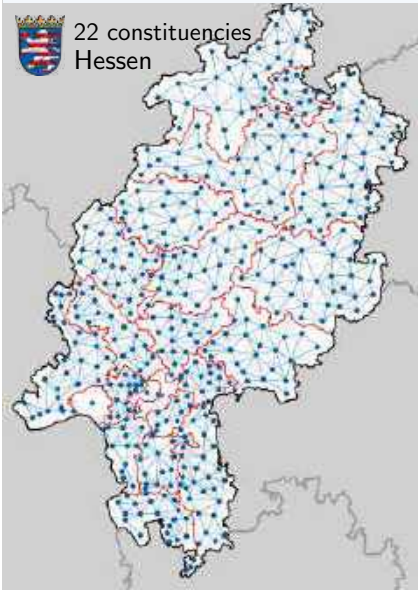
constraints for each constituency:

- **ensure** connectivity
- **ensure** population deviation $\leq 25\%$

objectives:

- **min** amount of population deviation
- **max** conformity with adm. boundaries
- **max** continuity

Network Model for the Political Districting Problem



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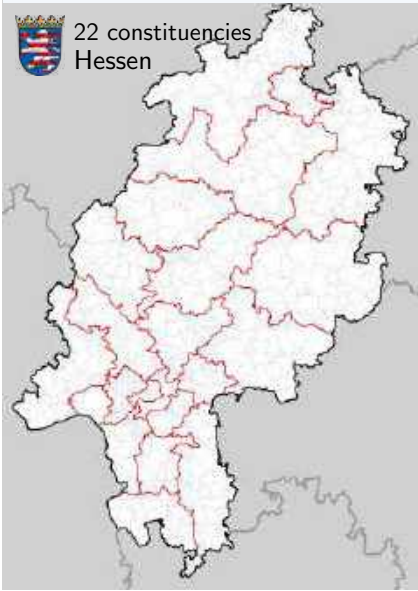

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Capacitated Connected Graph Partitioning

communities $c \in C$, districts $d \in D$, $|D| = 270$

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$x_{cd} \in \{0, 1\}$ community c in district d?

Capacitated Connected Graph Partitioning

communities $c \in C$, districts $d \in D$, $|D| = 270$

$$\sum_{d \in D} x_{cd} = 1 \quad \forall c \in C$$ assign every community

$$x_{cd} \in \{0, 1\}$$ community c in district d?

Capacitated Connected Graph Partitioning

communities $c \in C$, districts $d \in D$, $|D| = 270$

$$\sum_{d \in D} x_{cd} = 1 \quad \forall c \in C \quad \text{assign every community}$$

$$\sum_{c \in C} \text{pop}_c \cdot x_{cd} \qquad \forall d \in D \quad \text{pop limits}$$

$$x_{cd} \in \{0, 1\} \quad \text{community } c \text{ in district } d\text{?}$$

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districts connected, compact

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districts connected, compact

$$x_{cd} \in \{0, 1\}$$ community c in district d?

inzwischen in der Lehre angekommen . . .

- in der Wissenschaft ist immer das erste Mal schwer

- heute können (und müssen) unsere Studierenden den Ansatz „nachbauen“

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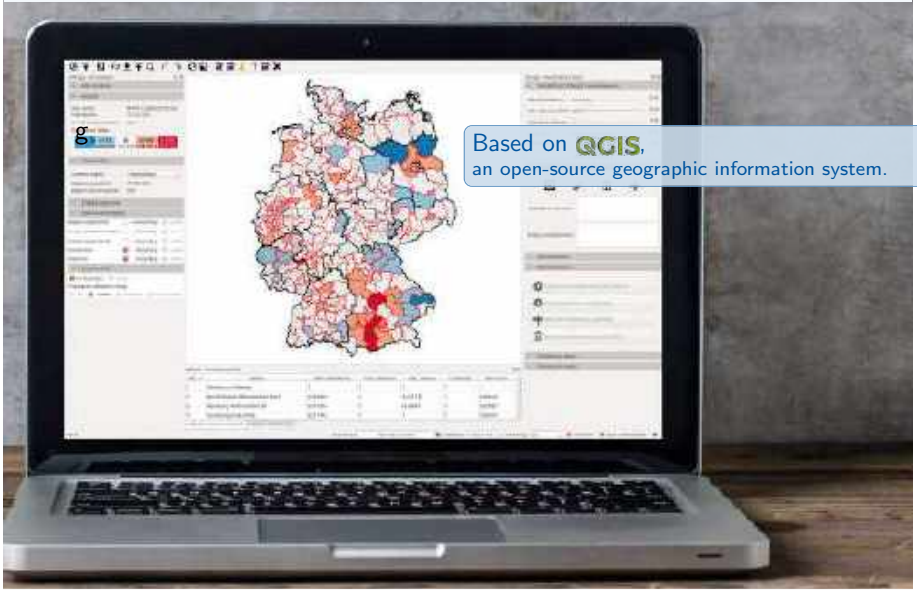
Our Research ready to use in a Decision Support System

Our Research ready to use in a Decision Support System



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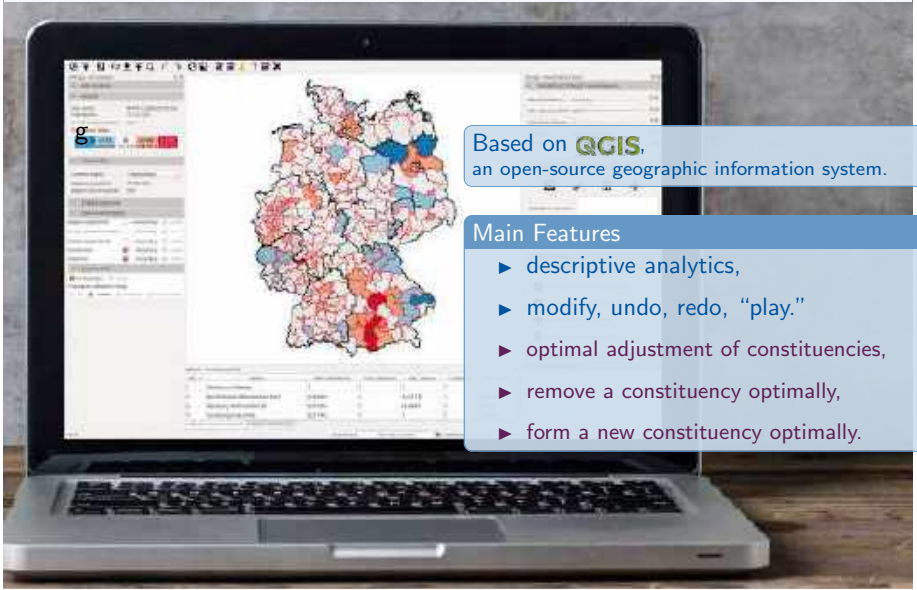


Based on QGIS,
an open-source geographic information system.

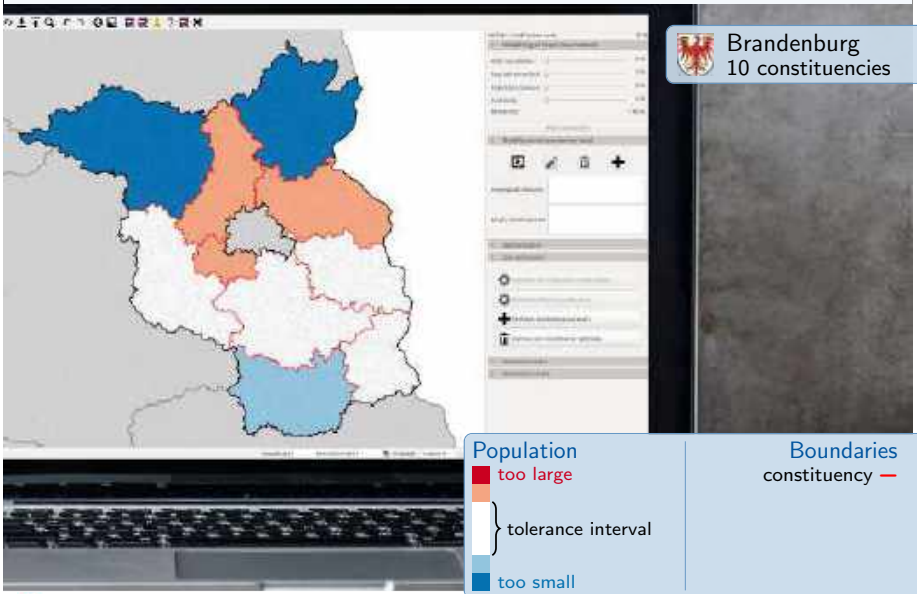
Main Features

- descriptive analytics,
- modify, undo, redo, "play."
- optimal adjustment of constituencies,
- remove a constituency optimally,
- form a new constituency optimally.

Example: Optimize 2013's Constituencies for 2017 Election

Brandenburg
10 constituencies

Example: Optimize 2013's Constituencies for 2017 Election

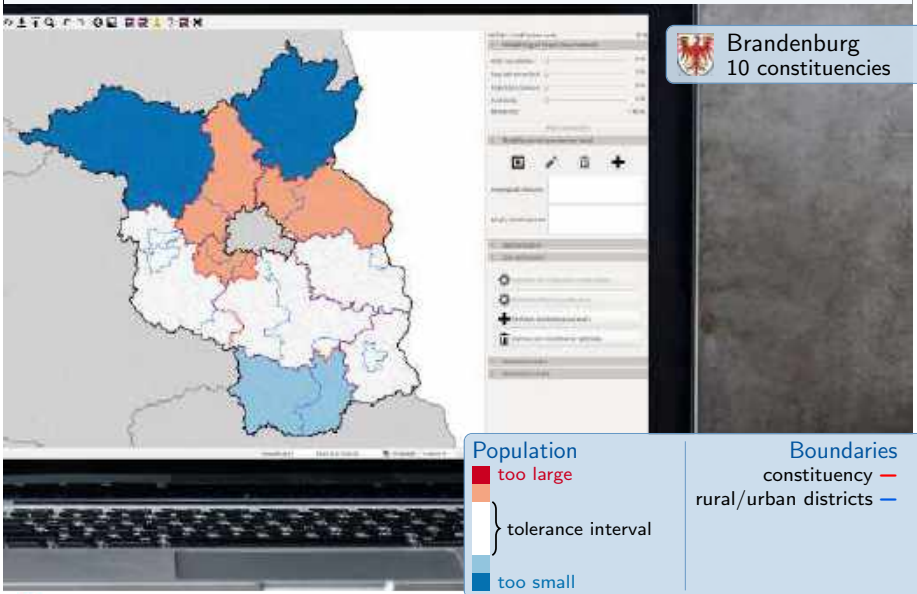


Brandenburg
10 constituencies

Population
too large
tolerance interval
too small

Boundaries
constituency —

Example: Optimize 2013's Constituencies for 2017 Election



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rural/urban districts

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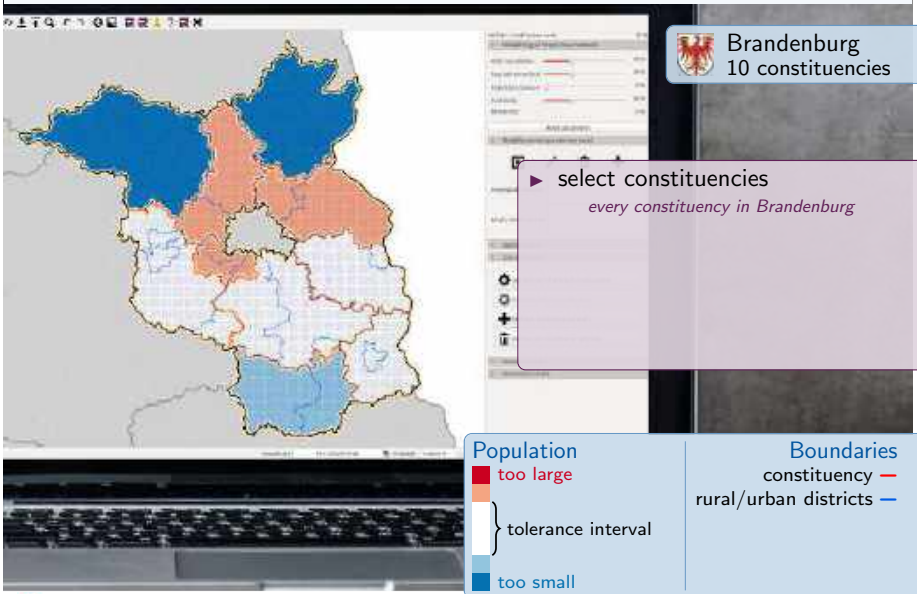


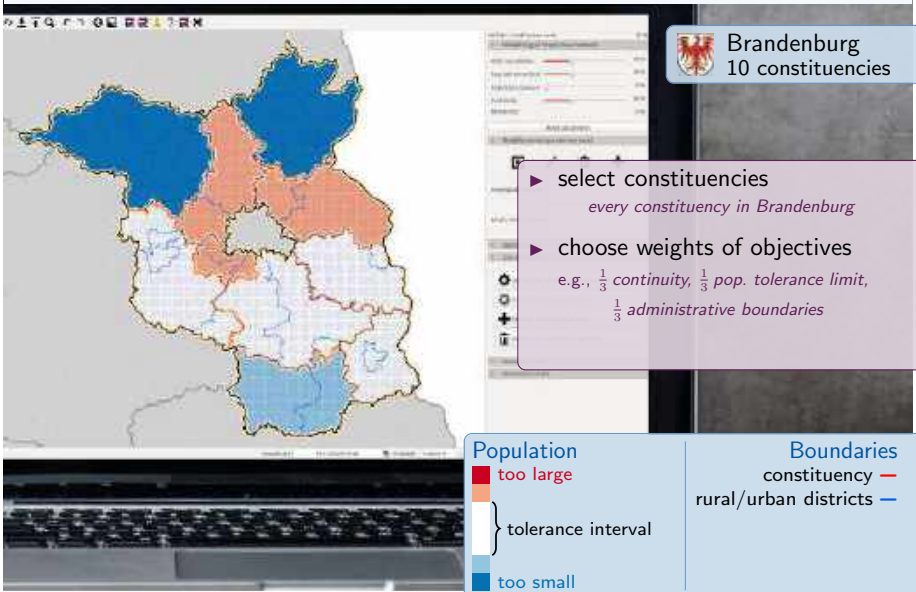
Brandenburg
10 constituencies

- select constituencies
  every constituency in Brandenburg

Population
- too large
- tolerance interval
- too small

Boundaries
- constituency
- rural/urban districts

Example: Optimize 2013's Constituencies for 2017 Election



Brandenburg
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- select constituencies
  every constituency in Brandenburg
- choose weights of objectives
  e.g., $\frac{1}{3}$ *continuity*, $\frac{1}{3}$ *pop. tolerance limit*, $\frac{1}{3}$ *administrative boundaries*

Population
too large
tolerance interval
too small

Boundaries
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rural/urban districts

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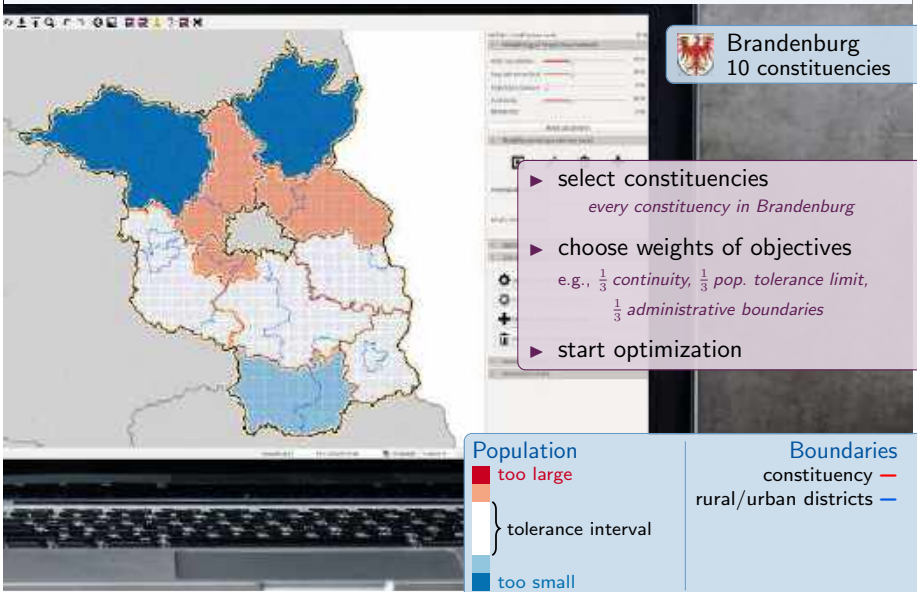

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Population
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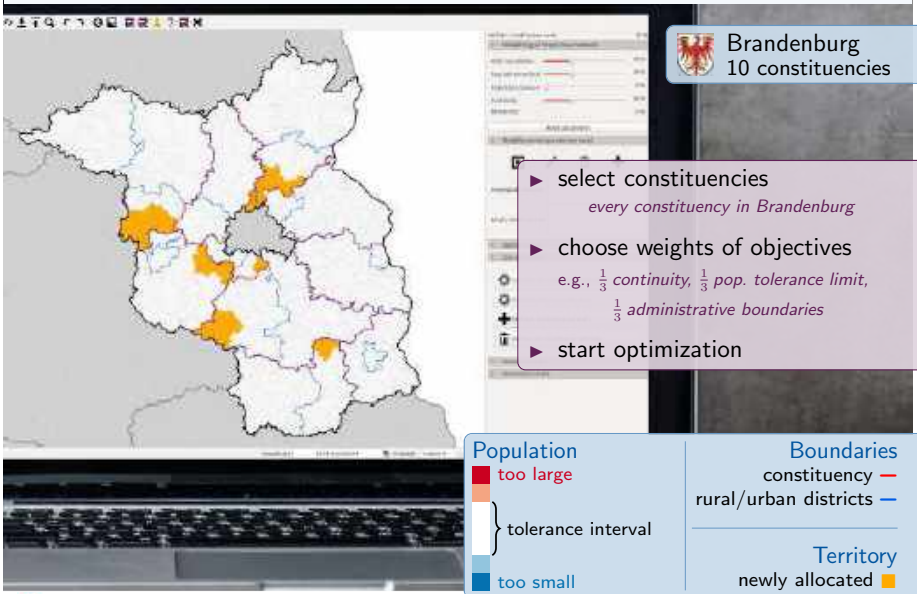

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Population
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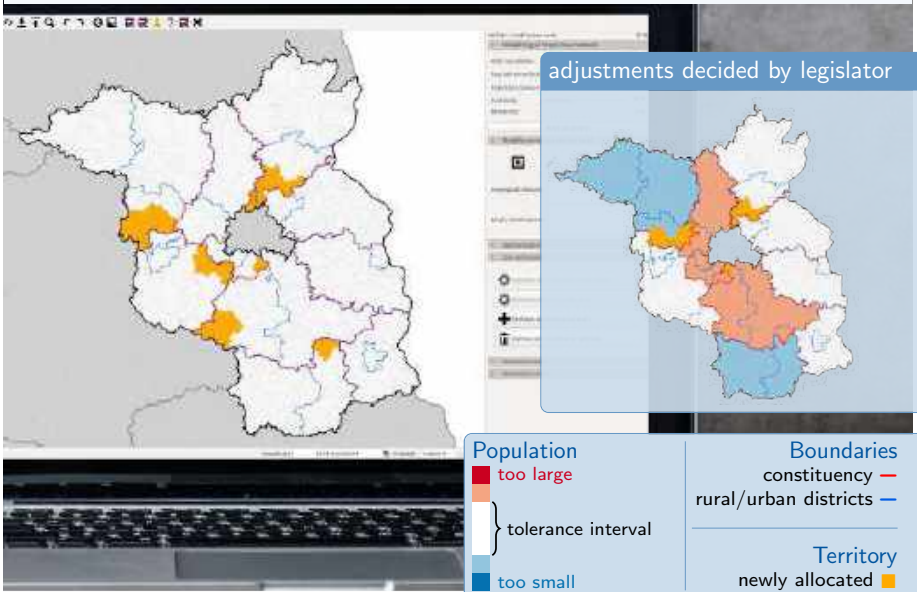
Boundaries
constituency
rural/urban districts

Territory
newly allocated

Example: Optimize 2013's Constituencies for 2017 Election



adjustments decided by legislator

Population
- too large
- tolerance interval
- too small

Boundaries
- constituency
- rural/urban districts

Territory
- newly allocated

Eine Disziplin alleine schafft es nicht . . .

- wir wissen nicht, ob das Wahlgesetz *richtig* oder *gut* ist // Randbedingungen
- wir wissen nicht, ob „one person, one vote“ erstrebenswert ist // Zielfunktion

- aber **wenn** wir uns geeinigt haben, dann erhalten wir eine Wahlkreiseinteilung, die unter diesen Rahmenbedingungen nicht besser sein **kann**

→ Transparenz

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Kurz innehalten

- kann man Wahlkreise einteilen, kann man andere Gebiete einteilen

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Kurz innehalten

- ein solches Planungswerkzeug ist auch ein Spielzeug $\rightarrow$ Fantasie!

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Was hat es gebracht?

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Was hat es gebracht?

2012–

Research Talk #6?

- über unsere Forschung haben wir wenig gesprochen

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Anregungen

#IchBinHanna

Anregungen

- Mathe rettet nicht die Welt

Anregungen

- Mathe rettet nicht die Welt, aber ohne retten wir die Welt auch nicht